INVARIANT CURVES FOR ENDOMORPHISMS OF $\mathbb{P}^1 \times \mathbb{P}^1$

Let $A_1, A_2 \in \mathbb{C}(z)$ be rational functions of degree at least two that are neither Lattès maps nor conjugate to $z^{\pm n}$ or $\pm T_n$. In the talk, we describe invariant, periodic, and preperiodic algebraic curves for endomorphisms of $\mathbb{P}^1 \times \mathbb{P}^1$ of the form $(z_1, z_2) \to (A_1(z_1), A_2(z_2))$. In particular, we show that if $A \in \mathbb{C}(z)$ is not a "generalized Lattès map", then any (A, A)-invariant curve has genus zero and can be parametrized by rational functions commuting with A. As an application, for A defined over a number field we give a criterion for a point of $\mathbb{P}^1 \times \mathbb{P}^1$ to have a Zariski dense (A, A)-orbit in terms of canonical heights, and deduce from this criterion a version of a conjecture of Zhang.