Le Monde de Monderer

Sergiu Hart

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Students

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- Irit Talmor
- Aner Sela
- Ilana Weismann
- Zeev Nutov
- Yaron Leitner
- Eyal Chermony
- Shlomit Hon-Snir
- Noa Kfir-Dahav
- Itai Ashlagi
- Raphael Paul Eidenbenz

Major Scientific Contributions

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- Values of non-atomic games

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- Common belief

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- Potential games

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Values of Non-Atomic Games

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  A priori evaluation of the expected outcome of the game

Values of Non-Atomic Games

Values of Non-Atomic Games

Main Theorem. *For every μ-symmetric, continuous linear operator $\psi : pNA(\mu) \to FA$ there exists a unique pair (f_0, g_0) in $L_\infty \times L_\infty$ s.t. for every $v \in pNA(\mu)$ and every $S \subseteq I$ the following holds:*

$$\psi v(S) = \int_0^1 \partial v(x, S) f_0(x)\, dx + \left(\int_0^1 \partial v(x, I) g_0(x)\, dx \right) \mu(S). \qquad (*)$$

The correspondence $(f_0, g_0) \leftrightarrow \psi$ defined in $(*)$ is a linear isomorphism between $L_\infty \times L_\infty$ and the space of μ-symmetric continuous linear operators from $pNA(\mu)$ into FA, and moreover:

(a) $\mathrm{Max}(\| f_0 \|_\infty, \| g_0 \|_\infty) \leq \| \psi \| \leq \| f_0 \|_\infty + \| g_0 \|_\infty$.

(b) ψ is positive iff $f_0 \geq 0$ and $g_0 \geq 0$.

(c) ψ satisfies the efficiency axiom iff $f_0 + g_0 = 1$.

(d) ψ satisfies the dummy axiom iff $g_0 = 0$.

(e) ψ satisfies the projection axiom iff $\int_0^1 g_0 = 0$ and $\int_0^1 f_0 = 1$. ∎

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"MEASURE-BASED VALUES OF NON-ATOMIC GAMES"

Mathematics of Operations Research 1986

Common Belief

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- What **IS** a good approximation?

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"APPROXIMATING COMMON KNOWLEDGE WITH COMMON BELIEFS" (with Dov Samet)
Games and Economic Behavior 1989

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More detailed:

$$u^i(s^i, s^{-i}) - u^i(t^i, s^{-i}) = P(s^i, s^{-i}) - P(t^i, s^{-i})$$

for every $i \in N$, $s^i, t^i \in S^i$ and $s^{-i} \in S^{-i}$.

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"POTENTIAL GAMES" (with Lloyd Shapley)
Games and Economic Behavior 1996

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Dynamics and Learning

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"FICTITIOUS PLAY PROPERTIES FOR GAMES WITH IDENTICAL INTERESTS"
(with Lloyd Shapley)
Journal of Economic Theory 1996

"BELIEF AFFIRMING IN LEARNING PROCESSES"
(with Dov Samet and Aner Sela)
Journal of Economic Theory 1997

Cooperative (Coalitional) Games

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"WEIGHTED SHAPLEY VALUES AND THE CORE"
(with Dov Samet and Lloyd Shapley)
International Journal of Game Theory 1992

Game Theory & Computer Science

Game Theory & Computer Science

- Mechanism Design and Auctions

"BUNDLING EQUILIBRIUM IN COMBINATORIAL AUCTIONS" (with Ron Holzman, Noa Kfir-Dahav, and Moshe Tennenholtz)
Games and Economic Behavior 2004

"A LEARNING APPROACH TO AUCTIONS" (with Shlomit Hon-Snir and Aner Sela)
Journal of Economic Theory 1998

- Mechanism Design and Auctions
- Implementation

"MONOTONICITY AND IMPLEMENTABILITY"
(with Itai Ashlagi, Mark Braverman,
and Avinatan Hassidim)
Econometrica 2010

Game Theory & Computer Science

- Mechanism Design and Auctions
- Implementation
- Mediators and Correlation

"STRONG MEDIATED EQUILIBRIUM"
(with Moshe Tennenholtz)
Artificial Inteligence 2009

Game Theory & Computer Science

- Mechanism Design and Auctions
- Implementation
- Mediators and Correlation
- Distributed Games

"DISTRIBUTED GAMES"
(with Moshe Tennenholtz)
Games and Economic Behavior 1999

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*** * * ENJOY IT! * * ***