ERGODIC THEORY AND BEYOND - ABSTRACTS

Contents

1. Tim Austin

Title: Positive sofic entropy without relatively Bernoulli factors.

Abstract. The classical Kolmogorov–Sinai entropy is an invariant of probability-preserving transformations. Much of the resulting theory was successfully extended to actions of discrete amenable groups by Ornstein, Weiss and others.

Lewis Bowen's more recent notion of sofic entropy extends the Kolmogorov–Sinai definition to actions of sofic groups, a much larger class introduced by Weiss. A range of natural questions concern how entropy and its consequences differ between the sofic setting the amenable one.

After reviewing a special case of sofic entropy for certain free-product groups, this talk will present a new example of an action of such a group. The example has positive sofic entropy, but has no splitting as a direct product involving a Bernoulli factor. This contrasts with the world of amenable-group actions, where many such splittings would be guaranteed by the weak Pinsker theorem. The new example is an algebraic action, and its analysis depends on (slight modifications of) results from the theory of random regular low-density parity-check codes.

This material is part of an ongoing joint project with Lewis Bowen, Brandon Seward and Christopher Shriver.

2. Nir Avni

Conjugacy width in uniform lattices.

Abstract. Given a group G and an element $g \in G$, we say that the conjugacy width of g is finite if any element in the normal subgroup generated by g is a product of a bounded number of conjugates of g. Finite conjugacy width in arithmetic groups is very much related to the notion of Bounded Generation. We show that the elements of $Spin_{x_1^2+\cdots+x_{20}^2}(\mathbb{Z}[1/2])$ have finite conjugacy widths (whereas Bounded Generation fails).

This is a joint work with Chen Meiri.

3. Vitaly Bergelson

Title: A dynamical approach to the Prime Number Theorem and disjointness of additive and multiplicative semigroup actions.

Abstract. We will discuss a new type of ergodic theorem which has among its corollaries numerous classical results from multiplicative number theory, including the Prime Number Theorem, a theorem of Pillai–Selberg and a theorem of Erdős–Delange. This ergodic approach leads to a new dynamical framework for a general form of Sarnak's Möbius disjointness conjecture which focuses on the "joint independence" of actions of $(\mathbb{N}, +)$ and $(\mathbb{N}, \times)$. The talk is based on recent joint work with Florian Richter.

4. Manfred Einsiedler

Quantitative recurrence in measure classification problems.

Abstract. We will outline instances of using quantitative recurrence in partial measure classification theorems for higher rank actions.

In particular this allows us in joint work with Elon Lindenstrauss to establish the positive entropy measure classification for rank two actions on irreducible quotients of $\mathrm{SL}_2(\mathbb{R})^d$.

5. Matt Foreman

Title: Classification and anti-classification in ergodic theory and smooth dynamics.

Abstract. Celebrated work of Ornstein showed that entropy is a complete numerical invariant for Bernoulli Shifts. Halmos and von Neumann showed that the eigenvalues of the Koopman operator for an ergodic measure preserving transformation is a complete invariant. This led to an extensive search for further classification results for ergodic measure preserving transformations.

Joint work of the speaker with Rudolph and Weiss showed that the general case is impossible: the equivalence relation of measure conjugacy is not even

Borel. Further joint work with Weiss showed that this is impossible even for C^∞-flows on the torus.

There is general theory of classifications of equivalence relations (due to Friedman, Stanley, Hjorth, Kechris, Gao and others) that give benchmarks for the complexity of equivalence relations. These ideas have been recently applied to Kakutani equivalence (Kunde and Gerber) and to smooth dynamics (in joint work with Gorodetski).

This talk survey's these new results and discuss a conjecture of Sabok that would imply that measure conjugacy is *the* maximally complicated equivalence relation induced by a Polish group action.

6. Rachel Greenfeld

Title: The structure of translational tilings.

Abstract. Translational tiling is a covering of a space using translated copies of some building blocks, called the "tiles", without any positive measure overlaps. What are the possible ways that a space can be tiled? In the talk, we will discuss the study of this question, and report on recent progress, joint with Terence Tao.

7. Yonatan Gutman

Title: Strongly isomorphic symbolic extensions for expansive topological flows.

Abstract. We prove that finite-dimensional topological flows without fixed points and having a countable number of periodic orbits, have the small flow boundary property. This enables us to answer positively a question of Bowen and Walters from 1972: Any expansive topological flow has a strongly isomorphic symbolic flow extension, i.e. an extension by a suspension flow over a subshift. Previously Burguet had shown this is true if the flow is assumed to be C^2-smooth.

Joint work with Ruxi Shi (Sorbonne).

8. Adam Kanigowski

Title: Distribution of prime orbits for some dynamical systems.

Abstract. For a given topological dynamical system (X,T) we are interested in the distribution of orbits of an initial condition $x \in X$ sampled at prime times. We will recall known results, main methods and discuss some recent progress. We will also discuss some other non-standard ergodic averages related to Möbius disjointness conjecture of Sarnak.

9. Zemer Kosloff

Title: (Stationary) Bernoulli shift factors for inhomogeneous systems.

Abstract. Joint work with Terry Soo. We will survey recent results on the model of nonsingular Bernoulli shifts, its relation to classical questions in smooth ergodic theory and discuss recent results on existence of (stationary) Bernoulli shift factors for systems which are not measure preserving and the stark difference between finitary and non-finitary factors.

10. Or Landesberg

Horospherical group actions and rigidity of infinite measures in higher rank.

Abstract. Horospherical group actions on homogeneous spaces exhibit remarkable rigidity, as first demonstrated by Furstenberg's proof of unique ergodicity of the horocycle flow on compact hyperbolic surfaces. Subsequent work by Dani, Veech, Margulis and Ratner led to a complete classification of all finite ergodic measures with respect to such actions. In contrast, much less is known regarding infinite ergodic Radon measures – a natural object to consider in the context of infinite volume homogeneous spaces. In this talk we will describe an infinite measure rigidity result for horospherical group actions on a certain family of homogeneous spaces of higher rank. As a consequence we derive a unique ergodicity type statement for quotients by Zariski dense Anosov subgroups. Based on joint work with Minju Lee, Elon Lindenstrauss and Hee Oh.

11. Mark Pollicott

Title: Estimating Lyapunov exponents for random matrix products.

Abstract. Given matrices $A_1, \cdots, A_k \in GL(d, \mathbb{R})$ ($d \geq 2$ and $k \geq 2$) and a probability vector $(p_1, \cdots, p_n)$ we can associate the (largest) Lyapunov exponent in the usual way by

$$\lambda := \lim_{N \to +\infty} \frac{1}{N} \sum_{i_1, \cdots, i_N} (p_{i_1, \cdots, i_n}) \log \|A_{i_1} \cdots A_{i_N}\|$$

In this talk I will discuss some practical methods to estimate the value of λ. I will also discuss a few applications which will help to motivate this problem.

12. Ariel Rapaport

Title: Dimension of self-affine measures in $\mathbb{R}^d$.

Abstract. A self-affine measure is a stationary measure for a random walk on $\mathbb{R}^d$ which is generated by finitely many contracting affine maps. Self-affine measures are among the most studied and well-known fractal objects. When $d = 1$ or 2 their dimension theory is relatively well understood. When $d \geq 3$ much less is known. I will present new results in higher dimensions.

13. Daren Wei

Title: Time change rigidity for unipotent flows.

Abstract. Two flows are said to be Kakutani equivalent if one is isomorphic to the other after time change, or equivalently if there are Poincare sections for the flows so that the respective induced maps are isomorphic to each other. Ratner showed that if $G = \mathrm{SL}(2, \mathbb{R})$ and Γ is a lattice in G, and if u_t is a one parameter unipotent subgroup in G then the u_t action on G/Γ equipped with Haar measure is loosely Bernoulli, i.e. Kakutani equivalent to a circle rotation. Thus any two such systems $(\mathrm{SL}(2, \mathbb{R})/\Gamma_i, u_t, m_i)$ are Kakutani equivalent to each other. On the other hand, Ratner showed that if $G = \mathrm{SL}(2, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R})$ and Γ is a reducible lattice, and u_t is the diagonally embedded one parameter unipotent subgroup in G, then $(G/\Gamma, u_t, m)$ is not loosely Bernoulli.

We show that in fact in this case and many other situations one cannot have Kakutani equivalence between such systems unless they are actually isomorphic.

This is a joint work with Elon Lindenstrauss.