Nash Equilibrium and Dynamics

Sergiu Hart

June 2008

Conference in Honor of John Nash's 80th Birthday

Opening Panel

NASH EQUILIBRIUM AND DYNAMICS

Sergiu Hart

Center for the Study of Rationality
Dept of Economics Dept of Mathematics
The Hebrew University of Jerusalem

hart@huji.ac.il
http://www.ma.huji.ac.il/hart

Nash equilibrium

nash equilibrium

Nash equilibrium

Nash equilibrium

John Nash, Ph.D. Dissertation, Princeton 1950

Nash equilibrium

EQUILIBRIUM POINT:

John Nash, Ph.D. Dissertation, Princeton 1950

Nash equilibrium

EQUILIBRIUM POINT:

"Each player's strategy is optimal against those of the others."

John Nash, Ph.D. Dissertation, Princeton 1950

Nash equilibrium

Nash equilibrium

- **NON-COOPERATIVE**

Nash equilibrium

- **NON-COOPERATIVE**

  "absence of coalitions, communication, and side-payments"

Nash equilibrium

- **NON-COOPERATIVE**

  "**absence of coalitions, communication, and side-payments**"

  - $\longrightarrow$ *"Nash Program"*: non-cooperative foundation and implementation of cooperative approaches

Nash equilibrium

- **NON-COOPERATIVE**

Nash equilibrium

- NON-COOPERATIVE
- "MASS-ACTION" INTERPRETATION

Nash equilibrium

- NON-COOPERATIVE
- "MASS-ACTION" INTERPRETATION
- "RATIONAL" INTERPRETATION

Nash equilibrium

- NON-COOPERATIVE
- "MASS-ACTION" INTERPRETATION
- "RATIONAL" INTERPRETATION

  "prediction of the behavior to be expected of rational playing the game"

Nash equilibrium

- NON-COOPERATIVE
- "MASS-ACTION" INTERPRETATION
- "RATIONAL" INTERPRETATION

  "prediction of the behavior to be expected of rational playing the game"

  "we need to assume the players know the full structure of the game

Nash equilibrium

- NON-COOPERATIVE
- "MASS-ACTION" INTERPRETATION
- "RATIONAL" INTERPRETATION

  "prediction of the behavior to be expected of rational playing the game"

  "we need to assume the players know the full structure of the game ... quite strongly a rationalistic and idealising interpretation"

Nash equilibrium

Nash equilibrium

If each player

Nash equilibrium

If each player

- knows his own payoff function

Nash equilibrium

If each player

- knows his own payoff function
- is rational

Nash equilibrium

If each player

- knows his own payoff function
- is rational
- knows the (pure) strategy choices of the others

Nash equilibrium

If each player

- knows his own payoff function
- is rational
- knows the (pure) strategy choices of the others

Then these choices constitute
a (pure) **NASH EQUILIBRIUM**

Nash equilibrium

If each player

- knows his own payoff function
- is rational
- knows the (pure) strategy choices of the others

Then these choices constitute
a (pure) **NASH EQUILIBRIUM**

Aumann and Brandenburger 1995

Dynamics

Dynamics

FACT:

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"***

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games
(rather than: for specific classes of games)

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games
- ***"leading to Nash equilibrium"***

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games
- ***"leading to Nash equilibrium"*** : at a Nash equilibrium (or close to it) from some time on

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games
- ***"leading to Nash equilibrium"*** : at a Nash equilibrium (or close to it) from some time on
- ***"natural"***

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games
- ***"leading to Nash equilibrium"*** : at a Nash equilibrium (or close to it) from some time on
- ***"natural"*** :
  - *not* of the "exhaustive search" variety

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games
- ***"leading to Nash equilibrium"*** : at a Nash equilibrium (or close to it) from some time on
- ***"natural"*** :
  - *not* of the "exhaustive search" variety
  - simple, efficient (time, computation, ...)

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

- ***"general"*** : in all games
- ***"leading to Nash equilibrium"*** : at a Nash equilibrium (or close to it) from some time on
- ***"natural"*** :
  - *not* of the "exhaustive search" variety
  - simple, efficient (time, computation, ...)
  - ***"uncoupled"***

UNCOUPLED DYNAMICS

Uncoupled dynamics

UNCOUPLED DYNAMICS :

Each player knows ***only*** his own payoff function

Hart and Mas-Colell 2003

Uncoupled dynamics

UNCOUPLED DYNAMICS :

Each player knows ***only*** his own payoff function
(does *not* know the others' payoff functions)

Hart and Mas-Colell 2003

Uncoupled dynamics

UNCOUPLED DYNAMICS :

Each player knows ***only*** his own payoff function
(does *not* know the others' payoff functions)

- impossibility results

Hart and Mas-Colell 2003, 2006

Uncoupled dynamics

UNCOUPLED DYNAMICS :

Each player knows ***only*** his own payoff function
(does *not* know the others' payoff functions)

- impossibility results
- how long? exponential time

Hart and Mansour 2008

Dynamics

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

RESULT:

Dynamics

FACT:

There are no general, natural dynamics leading to Nash equilibrium

RESULT:

There cannot be general, natural dynamics leading to Nash equilibrium

Correlated equilibrium

Correlated equilibrium

CORRELATED EQUILIBRIUM

Aumann 1974

Correlated equilibrium

CORRELATED EQUILIBRIUM :

Nash equilibrium when players receive payoff-irrelevant information before the game

Aumann 1974

Correlated equilibrium

CORRELATED EQUILIBRIUM :

Nash equilibrium when players receive payoff-irrelevant information before the game

- ***There are general, natural dynamics leading to correlated equilibria***

Correlated equilibrium

CORRELATED EQUILIBRIUM :

Nash equilibrium when players receive payoff-irrelevant information before the game

- ***There are general, natural dynamics leading to correlated equilibria***

- **"Law of Conservation of Coordination"**

Correlated equilibrium

CORRELATED EQUILIBRIUM :

Nash equilibrium when players receive payoff-irrelevant information before the game

- ***There are general, natural dynamics leading to correlated equilibria***

- **"Law of Conservation of Coordination"**:

  There must be some **"coordination"** –
  either in the solution concept,
  or in the dynamic

HAPPY BIRTHDAY, JOHN!