FORCING EXERCISES
DAY 12

Problem 1. *Let $\mathbb{P} \in M$ be a separative partial order, and Γ the canonical name for the generic filter. Show that $q \Vdash \check{p} \in \Gamma$ if and only if $q \leq p$.*

Problem 2. *Let $\mathbb{P}$ be a poset, and let $\mathbb{Q}$ be a dense set of $\mathbb{P}$, viewed as a subposet. Do the following.*

(1) *If G is $\mathbb{P}$-generic, then if we set $H = G \cap Q$, we have that H is $\mathbb{Q}$-generic. Further, $G = \{p \in \mathbb{P} : (\exists q \in H) q \leq p\}$.*
(2) *If H is $\mathbb{Q}$-generic, then if we set $G = \{p \in \mathbb{P} : (\exists q \in H) q \leq p\}$ then G is $\mathbb{P}$-generic. Further, $H = G \cap Q$.*
(3) *If G and H are taken as in either of the two above, then $M[G] = M[H]$.*
(4) *Any $\mathbb{Q}$-name is a $\mathbb{P}$-name, and if $\tau_1, \ldots, \tau_n \in M^{\mathbb{Q}}$ then $\Vdash_{\mathbb{P}} \phi(\tau_1, \ldots, \tau_n)$ if and only if $\Vdash_{\mathbb{Q}} \phi(\tau_1, \ldots, \tau_n)$.*
(5) *For any $\tau \in M^{\mathbb{P}}$ there is a $\sigma \in M^{\mathbb{P}}$ such that $\Vdash_{\mathbb{P}} \tau = \sigma$.*

Problem 3. *Two forcing notions $\mathbb{P}$ and $\mathbb{Q}$ are forcing equivalent if there are $\tau \in \mathbb{P}$ and $\sigma \in \mathbb{Q}$ such that*

(a) *$\Vdash_{\mathbb{P}} \tau$ is a $\mathbb{Q}$-generic filter.*
(b) *$\Vdash_{\mathbb{Q}} \sigma$ is a $\mathbb{P}$-generic filter.*
(c) *If G is $\mathbb{P}$-generic then $G = \sigma[\tau[G]]$.*
(d) *If H is $\mathbb{Q}$-generic then $H = \tau[\sigma[H]]$.*

Do the following:

(1) *Show that if $\mathbb{Q}$ is a dense subset of $\mathbb{P}$, then the two are forcing equivalent.*
(2) *Show that forcing equivalence is an equivalence relation.*

Problem 4. *Revisit Problems 3 and 4 from yesterday.*

Problem 5. *Let $\mathbb{P}$ be countably closed forcing and let T be a tree of height ω_1 with no cofinal branch. Show that for all M-generic G, T has no cofinal branch in $M[G]$.*

Problem 6. *Let $\mathbb{P}$ be countably closed forcing and $2^\omega = \omega_2$. Suppose that T is an ω_2-tree. Show that if $b \in M[G]$ is a cofinal branch through T, then $b \in M$.*