FORCING EXERCISES
DAY 6

Problem 1. *Let κ be a regular cardinal. Construct a tree of height κ with no cofinal branch.*

Definition 1. *A tree T is normal if for all $\alpha < \beta < \mathrm{ht}(T)$ and for all $x \in \mathrm{Lev}_\alpha(T)$, there is a $y \in \mathrm{Lev}_\beta(T)$ with $x <_T y$.*

Problem 2. *Let κ be a regular cardinal. Construct a* normal *tree of height κ with no cofinal branch.*

Problem 3 (*)**.** *Let κ be a regular cardinal. Show that every κ-tree has a normal subtree.*

Definition 2. *Let $(T, <_T)$ be a partially ordered set. T is* splitting *if for every $x \in T$ there are $y_0, y_1 \in T$ such that $x <_T y_0$, $x <_T y_1$, and there is no $z \in T$ such that $y_0, y_1 \leq_T z$.*

Problem 4. *Let κ be a regular cardinal. Suppose that T is a normal κ-tree with no cofinal branch. Show that T is splitting.*

Problem 5. *Show that if T is a special ω_1-tree, then T has no cofinal branch.*

Problem 6. *Show that if T is a special ω_1-tree, then T is not Suslin.*

Problem 7 (*)**.** *Let κ be an uncountable cardinal. Suppose that T is a tree of height κ^+ with levels of size less than κ. Show that T has a cofinal branch.*

Theorem 1 (Finite Ramsey Theorem)**.** *For every $k < \omega$ there is $n < \omega$ such that for every $\chi : [n]^2 \to 2$ there is a monochromatic set of size k.*

Problem 8. *Use König Infinity Lemma and Infinite Ramsey Theorem to prove Finite Ramsey Theorem.*