FORCING EXERCISES
DAY 3

Problem 1. *Given an example of a poset for which the antichains are exactly the almost disjoint families of subsets of* ω*.*

Problem 2. *Recall the definition of* open *in a poset. Show that* $\mathrm{MA}(\kappa)$ *is equivalent to the same statement where the word 'dense' is replaced with the words 'dense open'.*

Problem 3. *An antichain is* maximal *if it is maximal under containment. Show that* $\mathrm{MA}(\kappa)$ *is equivalent to the formulation where 'dense sets' is replaced with 'maximal antichains'.*

Definition 1. *Let* $\mathbb{P}$ *be a poset.*

(1) $\mathbb{P}$ *is* σ*-centered if there is a partition of into set* P_i *for* $i < \omega$ *such the for all* i *and all* $p, q \in P_i$*,* p *and* q *are compatible.*
(2) $\mathbb{P}$ *is* $\aleph_1$*-Knaster if for every sequence* $\langle p_\alpha \mid \alpha < \omega_1 \rangle$ *of elements of* $\mathbb{P}$*, there is an unbounded* $I \subseteq \omega_1$ *such that for all* $\alpha, \beta \in I$*,* p_α *is compatible with* p_β*.*

Problem 4. *Show that for a poset* $\mathbb{P}$*,* σ*-centered implies* $\aleph_1$*-Knaster implies ccc.*

Definition 2. *Given a poset* $\mathbb{P}$*,* $\mathrm{cc}(\mathbb{P})$ *is the least cardinal* κ *such that* $\mathbb{P}$ *has no antichains of size* κ*.*

Problem 5 (*). *Suppose that* $\mathbb{P}$ *is a poset such that* $\mathrm{cc}(\mathbb{P}) > n$ *for all* $n < \omega$*. Show that* $\mathrm{cc}(\mathbb{P}) > \omega$*, ie show that* $\mathbb{P}$ *has an infinite antichain.*

Problem 6 (**). *Let* $\mathbb{P}$ *be a fixed poset. Show that* $\mathrm{cc}(\mathbb{P})$ *is a regular cardinal.*