Problem 1. *Let κ, λ and μ be cardinals.*

(1) *Show that* $(\kappa^\lambda)^\mu = \kappa^{(\lambda\cdot\mu)}$.
(2) *Show that* $\kappa^\kappa = 2^\kappa$.

Problem 2. *Let A be an infinite set of size κ. Show that the set of all bijections from A to A has size 2^κ.*

Problem 3. *Let κ and λ be cardinals with $\lambda \leq \kappa$ and κ infinite. We write $[\kappa]^\lambda$ for the collection of all subsets of κ of size λ. Show that the cardinality of $[\kappa]^\lambda$ is κ^λ.*

Problem 4. *Show that the supremum of a set of cardinals is a cardinal.*

Problem 5. *Suppose that $\langle \alpha_i \mid i < \lambda \rangle$ is an increasing sequence of ordinals cofinal in some cardinal κ. Show that* $\mathrm{cf}(\kappa) = \mathrm{cf}(\lambda)$.

Problem 6 (*)**.** *Without using the Axiom of Choice show that there is a surjection from $\mathcal{P}(\omega)$ to ω_1.*

Problem 7 (*)**.** *Let κ be an infinite cardinal. Construct a family $\mathcal{F}$ of size κ of functions from κ^+ to κ^+ such that for all $\alpha, \beta < \kappa^+$ there is a function $f \in \mathcal{F}$ such that either $f(\alpha) = \beta$ or $f(\beta) = \alpha$.*

Problem 8 (*)**.** *Define a set $S \subseteq \omega_1 \times \omega$ to be a* large rectangle *if $S = A \times B$ where A is uncountable and B is infinite. Show that* CH *implies that there is a set $T \subseteq \omega_1 \times \omega$ such that for all large rectangles S, $T \cap S \neq \emptyset$ and $T \cap ((\omega_1 \times \omega) \setminus S) \neq \emptyset$.*

Problem 9 (*)**.** *Let κ be an infinite cardinal and $\prec$ be a well-ordering of κ. Show that there is an $X \subseteq \kappa$ such that $|X| = \kappa$ and $\prec$ and $<$ agree on X.*