

Number Theory and Arithmetic, Jerusalem 2024 - Abstracts

Eyal Goren: Foliations and arithmetic of Shimura varieties

I will report on joint work with Ehud De Shalit (HUJI) concerning foliations on Shimura varieties in positive characteristic. After a brief general introduction to foliations, I will focus on the foliations we have defined and their connection to the geometry of Shimura varieties and their morphisms, as well as to modular forms. Time allowing, I will describe work in progress concerning the infinitesimal structure of foliations and integral subvarieties.

Amnon Besser: The theory of local indexes

Local (double or triple) indexes, are simple gadgets that generalize residues of one forms. I will survey uses of local indexes, two of which are related to work of de Shalit, on his p -adic regulator with Coleman and on the cup product formula for Mumford curves. I will also mention how these indexes can be obtained, and probably generalized, using ideas of Vologodsky.

Henri Darmon: The class number one problem for real quadratic fields. (Joint with Elias Caeiro.)

The determination by Heegner, Baker and Stark of the complete list of imaginary quadratic fields of class number one relies critically on the theory of complex multiplication. A conjectural extension of this theory to real quadratic fields based on the notion of “rigid analytic elliptic cocycles” is shown to yield similar lists for some explicit families of real quadratic orders with small regulators. An essential ingredient in our calculations is De Shalit’s proof of the Mazur-Tate tame refinement of the exceptional zero conjecture of Mazur, Tate and Teitelbaum for elliptic curves of prime conductor, which Udi was working on when I first met him in Princeton in 1991.

Alex Lubotzky: Good locally testable codes

An error-correcting code is *locally testable* (LTC) if there is a random tester that reads only a small number of bits of a given word and decides whether the word is in the code, or at least close to it. A long-standing problem asks if there exists such a code that also satisfies the golden standards of coding theory: constant rate and constant distance. Unlike the classical situation in coding theory, random codes are not LTC, so this problem is a challenge of a new kind. We construct such codes based on what we call (*Ramanujan*) *Left/Right Cayley square complexes*. These objects seem to be of independent group-theoretic interest. The codes built on them are 2-dimensional versions of the expander codes constructed by Sipser and Spielman (1996). The main result and lecture will be self-contained. But we hope also to explain how the seminal work of Howard Garland (1972) on the cohomology of quotients of the Bruhat-Tits buildings of p -adic Lie group has led to this construction (even though it is not used at the end). Based on joint work with I. Dinur, S. Evra, R. Livne, and S. Mozes.

Uri Bader: Do not disturb the circle - a modern tribute to ancient mathematics

Alternative title: On the sum of angles in a polygon - a personal tribute to Ron and Udi, celebrating their contribution to math education in Israel. I will discuss a theorem of Euclid from antiquity, Fenchel’s take on it from 1929 and a conjecture of Borsuk from 1947, aka the Fary-Milnor theorem. The talk will be accessible to high school students and, as any talk should, it will contain a proof, a joke, a lie and a dream.