

# The Mathematics of General Relativity

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## A – Special relativity

1. The Minkowski space: the Lorentz metric, geodesics and null geodesics.
2. The Poincaré group: the affine Lorentz group, the group of isometries of the Lorentz metric. Analysis of its subgroups, interpretation in terms of "frame of reference".
3. Classification of the orbits of the Poincaré group: classical models of "elementary particles". Introduction of "Spin", "Mass", "Helicity" for massless particles: the "photons". Description of the symplectic structure.
4. Interpretation of some orbits in terms of space of geodesics or null geodesics.

## B – General relativity

5. Lorentz manifolds. Pseudo-riemannian metrics. Geodesics, Levi-Civita connexion.
6. The "principle of general covariance": the "Energy-Momentum tensor" as a 1-form on the quotient of the space of Lorentz metrics by the action of the diffeomorphisms with compact support. The "Eulerian distributions", models for "matter" in "space-time": tensor valued distributions on space-time satisfying "Euler's conservation equations", regarded as a condition of invariance under the group of diffeomorphisms of space-time.
  - Geodesics.
  - Strings.
  - Shells.
  - Continuous media.
7. The Einstein Equation.
8. A special solution of the Einstein equation: the Shwartzild solution.
9. A global model of universe: The Friedman-Lemaître solution.

# Short bibliography

## Books

**Albert Einstein,**

- Relativity: The Special and the General Theory (Amazon.com) [en]

**Jean-Marie Souriau,**

- Géométrie et Relativité, Hermann Ed. 1964. [fr]

**John Wheeler,**

- "Gravitation", W. H. Freeman, San Francisco, 1973. [en]

**Hermann Weyl,**

- Space, Time, Matter (Amazon.com) [en]

## Research Papers

**Patrick Iglesias & Jean-Marie Souriau,** Heat Cold and Geometry, Differential Geometry and Mathematical Physics, Cahen & als. edit., Reidel Publishing Company, 1983, Paris. [en] — On the web at: <http://math.huji.ac.il/~piz/documents/HCG.pdf>

**Jean-Marie Souriau,** Modèle de particule à spin dans le champ électromagnétique et gravitationnel, Ann. Inst. Henri Poincaré, 1974, Paris [fr].

## Notes on the course

Note on the course, exercises, more bibliography will be found with time on

- <http://math.huji.ac.il/~piz/Site/General%20Relativity.html>

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