Amitsur Memorial Symposium 2024 - Abstracts

June 16, 2024

Shlomo Gelaki - Twisted unipotent groups

Let G be a unipotent complex algebraic group, and let $O(G)$ be its (affine commutative Hopf) algebra of polynomial functions. Let J in $(O(G) \otimes O(G))^*$ be a Hopf 2-cocycle for G. The twisting procedure of Drinfeld yields a new cotriangular Hopf algebra structure $(O(G)^J, R^J)$ on the underlying coalgebra of $O(G)$ by using J to deform the standard multiplication and R-form of $O(G)$.

In my talk I will first discuss the bijection between Hopf 2-cocycles for G (equivalently, tensor structures on the forgetful functor Rep(G) —> Vec, where Rep(G) is the category of finite dimensional rational representations of G) and pairs (h, ω), where h is a Lie subalgebra of Lie(G) and $\omega \in \bigwedge^2 h^*$ is a non-degenerate 2-cocycle.

Next I will discuss the algebraic structure and representation theory of the Hopf algebras $O(G)^J$ and the related one-sided twisted function algebras $O(G)_J$.

Be'eri Greenfeld - Local smallness and global largeness: a quantitative approach

The Kurosh Problem asks whether a finitely generated algebraic algebra can be infinite-dimensional. This is the algebraic counterpart of the Burnside Problem, which asks whether a finitely generated periodic group can be infinite. We examine these problems from modern perspectives, including quantitative asymptotics, free subobjects and randomization.

Danny Neftin - Rational connectedness and parametrization of Galois extensions

Given two Galois extensions with group G over the field of rationals $\mathbb{Q}$, is there a Galois extension with group G over the rational function field $\mathbb{Q}(t)$ that specializes to both extensions? This and more general rational connectedness questions lie at the heart of the geometry of parametrizing spaces (or versal torsors) for Galois extensions with group G over $\mathbb{Q}$. We shall discuss recent advances in understanding the number of equivalence classes under rational connectedness on such spaces with emphasis on the case of dihedral groups G of 2-power order, where little is known about the rationality of the corresponding parametrizing spaces. Joint work with D. Krashen

Elad Paran - Some aspects of quaternionic algebraic geometry

We shall review recent works concerned with foundational aspects of quaternionic algebraic geometry:

1. A quaternionic nullstellensatz for the ring R of polynomials in n central variables over the quaternion algebra $\mathbb{H}$, in both abstract and explicit form (a recent result due to Aryapoor).

2. The following theorem about the geometry of zero sets of polynomials in R: If a polynomial vanishes on all common zeros with commuting coordinates of a left ideal J in R , then it vanishes on all common zeros of J in $\mathbb{H}^n$. This confirms a recent conjecture of Gori, Sarfatti and Vlacci.

Joint work with Gil Alon.

Zinovy Reichstein - Hilbert's 13th Problem for algebraic groups

The algebraic form of Hilbert's 13th Problem asks for the resolvent degree $rd(n)$ of the general polynomial $f(x) = x^n + a_1x^{n-1} + ... + a_n$ of degree n, where $a_1, ..., a_n$ are independent variables. Here $rd(n)$ is the minimal integer d such that every root of $f(x)$ can be obtained in a finite number of steps, starting with C(a_1, ..., a_n) and adjoining an algebraic function in $\leq d$ variables at each step. It is known that $rd(n) = 1$ for every $n \leq 5$. It is not known whether or not $rd(n)$ is bounded as n tends to infinity; it is not even known whether or not $rd(n) > 1$ for any n. Recently Farb and Wolfson defined the resolvent degree $rd_k(G)$, where G is a finite group and k is a field of characteristic 0. In this setting $rd(n) = rd_{\mathbb{C}}(S_n)$, where S_n is the symmetric group on n letters and $\mathbb{C}$ is the field of complex numbers. In this talk I will define $rd_k(G)$ for any field k and any algebraic group G over k. Surprisingly, Hilbert's 13th Problem simplifies when G is connected. My main result is that $rd_k(G) \leq 5$ for an arbitrary connected algebraic group G defined over an arbitrary field k. If time permits, I will also discuss recent joint work with Oakley Edens on Hilbert's 13th Problem (this time, the original version, for the group $G = S_n$) in prime characteristic.

David Saltman - Stable Rationality and Cyclicity

There are two open questions about prime degree division algebras that are of interest. One is whether they are all cyclic. The other concerns the center $Z(F,p)$ of the generic division algebra of degree p written $UD(F,p)$. Specifically, is $Z(F,p)/F$ stably rational? We trace a connection between theses questions by showing the following. Let F be a field of characteristic 0 containing a primitive p root of one for p an odd prime. We outline the argument that if $Z(F,p)$ is NOT stably rational then $UD(F,p)$ is NOT cyclic.

Yoav Segev- Nonassociative algebras having semisimple indempotents

This talk deals with certain nonassociative, not necessarily commutative algebras possessing semisimple idempotents with specific behavior generalizing Jordan type. Specific enough that, on the one hand, they render an interesting (once you start) classification of the algebras, but on the other hand too specific to find their way into general questions about axial algebras.

This talk is dedicated to a very dear person, Avinoam Mann. This is joint work with Louis Rowen.

The organizers forbade me to write about the current situation in Israel. Due to my great respect to the late Professor Mann, I do not withdraw my talk.

Sara Westreich - Remembering Miriam Cohen: Reflections on her and our mathematical collaboration

I will briefly describe Mia's work over the years, from the 70s until quite recently. From our joint work I choose to focus on the term "Conjugacy classes for Hopf algebras". We first extended the notion of conjugacy classes and class sums from finite groups to semisimple Hopf algebras H and show that the conjugacy classes are obtained from the factorization of the Hopf algebra as irreducible left modules over its Drinfeld double $D(H)$. We then proved a list of results about character tables for semisimple Hopf algebras which extend ideas that occur in the representation theory of finite groups. For example, we can recover information about normal left coideal subalgebras of H (the analogues of normal subgroups and kernel of representations for groups) from its character table. We discussed various commutators and solvable Hopf algebras. In our last work, we showed how all irreducible modules over $D(H)$ are related to the conjugacy classes of H. We discussed examples of modules over the double of finite groups.