MATH 200 - SEC 104 - 2012W1

Midterm no. 3

1-1:50pm, Oct 29, 2012

Name (Last, First):__________________Ohi__________________

Student I.D. Number:____________________________________

Signature: ____

INSTRUCTIONS: This is a closed-book exam. You may not use any books, notes, papers, calculators, or other aids. Do all work on the sheets provided. There is an extra sheet on the back for scratch work. If you need an extra sheet, raise your hand and one will be provided. If you need more space for your solution, use the back of the sheets and leave an arrow for the grader. Please draw a box around your final answer.

There are 3 questions. Explain all your answers. Good Luck!

1. (20pts) Let $F(x, y) = 2x^4 + y^2 - x^2y^2$.

(a) Find all critical points of F and for each write whether it is a local minimum, local maximum or neither (if the method we use are inconclusive, just write so).

(b) Let $D = \{(x, y) \mid -1 \leq y \leq 3 - x^2\}$. Find the absolute minimum and maximum of F in D.

(c) Using part (b), what additional information can you deduce about the critical points from part (a)? Explain.

(a) $F_x = 8x^3 - 2xy^2$

$F_y = 2y(1 - x^2)$

$F_y = 0 \Rightarrow y = 0$ or $x = 1$ or $x = -1$

$y = 0 \Rightarrow 8x^3 = 0 \Rightarrow x = 0$ $(0,0)$ is a CP

$x = 1 \Rightarrow 8 - 2y^2 = 0 \Rightarrow y = \pm 2$ $(1,2)$ $(1,-2)$ are CP

$x = -1 \Rightarrow -8 + 2y^2 = 0 \Rightarrow y = \pm 2$ $(-1,2), (-1,-2)$ are CP

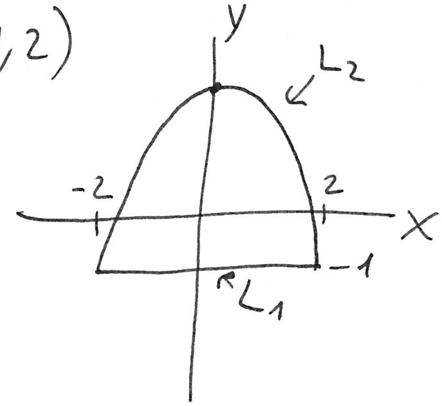
$F_{xx} = 24x^2 - 2y^2$

$F_{xy} = -4xy$

$F_{yy} = 2 - 2x^2$

	$(0,0)$	$(1,2)$	$(1,-2)$	$(-1,2)$	$(-1,-2)$
F_{xx}	0	16	16	16	16
F_{xy}	0	-4	4	4	-4
F_{yy}	2	0	0	0	0
D	0	-16	-16	-16	-16
point is	inconclusive	neither	neither	neither	neither

(b) CP in D are $(0,0)$ $(\pm 1, 2)$



in L_1 $y=-1$ $-2 \le x \le 2$

$g(x) = F(x,-1) = 2x^4 + 1 - x^2$

$g'(x) = 8x^3 - 2x = 0$

$\Updownarrow$

$x=0$ or $x = \pm\frac{1}{2}$

$(0,-1)$ $(\pm\frac{1}{2}, -1)$ are candidates.

in L_2 $y = 3-x^2$ $-2 \le x \le 2$

$$g(x) = F(x, 3-x^2) = 2x^4 + (3-x^2)^2 - x^2(3-x^2)^2$$

$$= 2x^4 + (9 - 6x^2 + x^4)(1-x^2)$$

$$= 2x^4 + 9 - 9x^2 - 6x^2 + 6x^4 + x^4 - x^6$$

$$= -x^6 + 9x^4 - 15x^2 + 9$$

$g'(x) = -6x^5 + 36x^3 - 30x = 0$

$\Updownarrow$

$x=0$ or $-x^4 + 6x^2 - 5x = 0$ $\Rightarrow x^2 = \dfrac{-6 \pm \sqrt{36-20}}{-2} = 5$ or 1

$x=0$ or $x = \pm 1$ $(x = \pm\sqrt{5}$ out of D$)$

$(0,3)$ $(\pm 1, 2)$ are candidates.

the endpoints $(\pm 2, -1)$ are candidates.

	$F =$
$(0,-1)$	1
$(\pm\frac{1}{2}, -1)$	$\frac{1}{8} + 1 - \frac{1}{4} = \frac{7}{8}$
$(0,0)$	0 $\Leftarrow$ min
$(\pm 1, 2)$	$2 + 4 - 4 = 2$
$(0,3)$	9
$(\pm 2, -1)$	$32 + 1 - 4 = 29$ $\Leftarrow$ max

(c) $(0,0)$ is the absolute minimum and it is not on the boundary of D so it must be a local minimum.

2. (10pts) Let $F(x, y)$ be a function whose 2nd order Taylor polynomial around $(1, -1)$ is

$$T_2(x, y) = 4 + (x-1)^2 - 2(x-1)(y+1) + 3(y+1)^2 .$$

(a) Write an equation for the tangent plane of F at $(1, -1)$.

(b) Is the point $(1, -1)$ a local minimum of F, a local maximum of F or neither? Explain.

(a) the tangent plane is just the linear part of T_2 so its $z = 4$

(b) from the definition of T_2 we get that

$F_{xx}(1,-1) = 2$

$F_{xy}(1,-1) = -2$

$F_{yy}(1,-1) = 6$

So $D = 12 - 4 = 8 > 0$ and $F_{xx} > 0$

So $(1,-1)$ is a local minimum.

3. (10pts) Let $F(x, y, z)$ be differentiable at $(1, 1, 1)$ and let $\nabla F(1, 1, 1) = \langle 3, -2, 4\rangle$.

(a) Let $g(t) = F(1 + \ln(1 + t), e^{-t}, \cos(t))$. Find $g'(0)$.

(b) Is it possible that $F(x, y, x^2y^2) = 1$ for all x and y? Explain.

(a) $\nabla F(1,1,1) = \langle 3,-2,4 \rangle$ means that at $(1,1,1)$

$F_x = 3 \quad F_y = -2 \quad F_z = 4$

if we denote $h_1(t) = 1 + \ln(1+t)$

$h_2(t) = e^{-t}$

$h_3(t) = \cos(t)$

then the chain rule implies

$g'(0) = 3h_1'(0) - 2h_2'(0) + 4h_3'(0)$

$= 3 \cdot 1 - 2(-1) + 4 \cdot 0 = 5$

(b) Let $G(x,y) = F(x, y, x^2y^2)$.

By the chain rule we have

$G_x(1,1) = F_x(1,1,1) \cdot 1 + F_y(1,1,1) \cdot 0 + F_z(1,1,1) \cdot 2$

$= 3 + 8 = 11$

So $G(x,y) = 1$ cannot be true, since in that case we would have had $G_x(1,1) = 0$