

## IS THE MULTIDIMENSIONAL ODD SHIFT SOFIC?

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Interpret a configuration  $x \in \{0, 1\}^{\mathbb{Z}^d}$  as a percolation, i.e. as defining a subset of  $\mathbb{Z}^d$  (the 1's) which in turn induces a subgraph of  $\mathbb{Z}^d$ , where as edges we take points whose  $\ell^1$ -distance is 1 (this is the same as taking the Cayley graph of  $\mathbb{Z}^d$  with the standard generating set). We can now talk about the connected components of  $x$ .

Let  $X \subseteq \{0, 1\}^{\mathbb{Z}^d}$  be the set of configurations in which every finite component has even size, and  $Y$  the set of configurations of odd size. It is an exercise to show that  $X$  is sofic.

**Problem.** Is  $Y$  sofic?

I believe that it is not, but I have no idea how to prove it.