Syllabus, "Fractal geometry and dynamics"

The course will provide an introduction to fractal geometry and geometric measure theory. We will discuss applications to dynamics and metric number theory as time allows.

We will not follow any one textbook. Notes will be available from

$$math.huji.ac.il/\sim mhochman/courses/fractals-2012$$

Other good sources are

- Kenneth Falconer, *The geometry of fractal sets*, Cambridge 1985
- Kenneth Falconer, *Techniques in fractal geometry*, Wiley 1997
- Pertti Mattila, "Geometry of sets and measures in Euclidean spaces", Cambridge 1995
- Christopher Bishop and Yuval Peres, "*Fractal sets in probability and analysis*", preprint, currently it can be found online if you google the title.

The following list of subjects is a superset of the material. We will cover the first few (unstarred) subjects in detail. This should take roughtly half of the course. We will cover a selection of topics from the starred sections as time allows.

Basic definitions and examples

Minkowski and Hausdorff dimension

Mass distribution principle and Billingsley's lemma

Frostman's lemma

Product sets

Fractals constructed by iteration

Iterated function systems

Self-similar sets

Self-affine sets

Geometry of measures

- Differentiation and density theorems
- Hausdorff measures
- Local dimension of a measure

Projection theorems

- Marstrand projection theorem and extensions
- Absolute continuity of projections
- Projections at the critical dimension, Kenyon's theorem
- Bernoulli convolutions

Intersection theorems (*)

- General intersection theorems
- Schmidt games and sets of large intersection, badly approximable numbers
- Thickness, sums of badly approximable numbers
- Kakeya sets and the Kakeya conjecture
- Furstenberg sets

Thermodynamic formalism (*)

- Gibbs measures
- Regular Cantor sets

Local methods (*)

- Furstenberg homogeneous sets and galleries
- Intersections of $\times m$- and $\times n$-invariant sets
- Tangent measures and densities
- Cassels-Schmidt theorem on normal numbers in Cantor sets