Statement of Research Interest

Gábor Damásdi
gabor.damasdi@gmail.com
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My primary interest lies in the field of combinatorial and discrete geometry. In particular, I am interested in Helly-type theorems, partition theorems, selection lemmas and the connections between these problems.

During my years of study I had the opportunity to choose from a wide range of courses. From the beginning I was interested mainly in discrete areas, such as graph theory, algorithms and discrete optimization. The first research topic I have worked on was the Zarankiewicz problem. It is a classical problem in extremal graph theory: determine the maximum number of edges in a bipartite graph that does not contain $K_{s,t}$ as an induced subgraph. With my co-authors we found the exact value in some new cases.

I am also intrigued by new methods, where we connect different areas of mathematics to solve combinatorial problems. For example some combinatorial problems have been solved using equivariant methods from topology and others have been attacked using algebraic tools such as Alon's Nullstellensatz. I was surprised to find that probabilistic methods can be used in unexpected areas such as covering theorems and crossing numbers of graphs. I am fond of all kind of games, so I was amazed in my descriptive set theory course, where I learned to use the theory of infinite games to solve problems related to Borel sets. I would like to be part of a research group where I can study and develop tools like these.

Since programming also attracted me I have completed a bachelor degree in computer science. There I was fascinated by formal languages, complexity theory and logic. I have found that the connection between mathematics and programming is very interesting and I wish to combine my knowledge in these areas in my future research. I would be very interested in conducting research about combinatorial structures using computer programming.

Over the last few years my interest turned a bit toward geometry. I have taken courses in finite, combinatorial and discrete geometry. On my own initiative I have also learned about equipartition theorems, centerpoint theorems, selection lemmas and other related topics. I especially like these areas because they combine ideas from combinatorics, geometry and topology. One of my favourite open problems is the first selection lemma, where the exact bounds are still not known in higher dimensions.

I wrote my bachelor thesis about slope numbers of graphs with bounded degree. The slope number of a graph G is the smallest number k such that G has a straight-line drawing in the plane using only k distinct edge slopes. During my master years I chose to study discrete geometry in more depth. Currently I am writing my master's thesis on a Helly-type theorem. Helly's theorem is one of the fundamental theorems of discrete geometry. It deals with intersection patterns of convex sets and it has played a central role in the study of convex sets. An important generalization, the Colorful Helly Theorem was proved by Lovász. In this version we ask for additional structure on the sets and draw a similar conclusion.

Theorem 1 (Colorful Helly, Lovász [1]). *Let $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{d+1} \subset \mathbb{R}^d$ be nonempty families of compact convex sets in $\mathbb{R}^d$, and suppose that for any choice $C_1 \in \mathcal{C}_1, \ldots, C_{d+1} \in \mathcal{C}_{d+1}$ the intersection $\bigcap_{i=1}^{d+1} C_i$ is nonempty. Then for some i all the sets in the family $\mathcal{C}_i$ have a point in common.*

Another way of generalization is to create quantitative versions [2]. Instead of asking for nonempty intersection we could ask for intersections with big volume. As a conclusion the intersection of all sets will be not too small.

Theorem 2 (Quantitative Helly, Naszódi [3]). *Let $\mathcal{C}$ be a nonempty family of convex sets in $\mathbb{R}^d$ such that the volume of its intersection is $vol(\cap\mathcal{C}) > 0$. Then there is a subfamily G of $\mathcal{C}$ with $|G| \leq 2d$ and $vol(\cap G) \leq e^{d+1} d^{2d+\frac{1}{2}} vol(\cap\mathcal{C})$.*

It is natural to ask for a combination of these two generalization. Let $VR(n)$ denote the ratio of volumes of the d-dimensional regular simplex and its inscribed ball. I have shown the following version:

Theorem 3 (Quantitative Colorful Helly). *Let $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{d(d+3)/2} \subset \mathbb{R}^d$ be nonempty families of compact convex sets in $\mathbb{R}^d$, and suppose that for any choice $C_1 \in \mathcal{C}_1, \ldots, C_{d(d+3)/2} \in \mathcal{C}_{d(d+3)/2}$ the intersection $\bigcap_{i=1}^{d(d+3)/2} C_i$ has volume greater or equal 1. Then for some i the intersection of all the sets in the family $\mathcal{C}_i$ has a volume at least $VR(n)$.*

The theorem uses $d(d+3)/2$ color classes which might look surprising since usually we have $d+1$ or $2d$ color classes. This number comes from the use of Löwner-John ellipsoids. I hope to decrease the number of color classes to $2d$ which would be the best possible.

References

[1] Bárány, I. A generalization of carathéodory's theorem. *Discrete Mathematics 40* (1982), 141–152.

[2] Bárány, I., Katchalski, M., and Pach, J. Quantitative helly-type theorems. *Proceedings of the American Mathematical Society 82*, 1 (1982), 109–114.

[3] Naszódi, M. Proof of a conjecture of bárány, katchalski and pach. *Discrete and computational geometry 55* (2016), 243–248.