

ARE THERE FACTOR-UNIVERSAL CA?

MICHAEL HOCHMAN

There are a number of notions of universality that have been studied for cellular automata. Most of these involve either computation-theoretic universality (i.e. that, under suitable interpretation of configurations, the CA performs universal computation), or embedding universality (specifically, that given an arbitrary CA g , the given CA f acts on some subshift in the same manner as g , after space and time is rescaled appropriately).

There is a related question that is more natural from a dynamical perspective:

Problem. Is there a CA $f : \Sigma^{\mathbb{Z}^d} \rightarrow \Sigma^{\mathbb{Z}^d}$ such that every other CA $g : \Delta^{\mathbb{Z}^k} \rightarrow \Delta^{\mathbb{Z}^k}$ is a factor of f ? i.e. such that for every g as above there is a continuous onto map $\pi : \Sigma^{\mathbb{Z}^d} \rightarrow \Delta^{\mathbb{Z}^k}$ such that $\pi f = g\pi$?

Note that we do not require the factor maps π to commute with the spatial shifts even when this makes sense, since then entropy of the shift would be a restriction. Since there are multidimensional SFTs with infinite entropy, the entropy of the CA itself poses no obstruction.

I would guess that the answer is negative.

I have looked at a related problem, namely the existence of universal effective systems and universal subactions of SFTs [On universality in multidimensional symbolic dynamics. To appear in *Discrete and Continuous Dynamical Systems*]. In fact, there is such an object for \mathbb{Z} -subactions, i.e. there is an SFT $X \subseteq \Sigma^{\mathbb{Z}^d}$ so that the shift-action in direction e_1 on X factors onto the same subaction for every other SFT. However, the problem above is of a different nature. Note for example the presence of periodic points for any CA, whereas the universal SFT subaction has no periodic points. The presence of periodic points means that one cannot get any information (or obstructions) from the recursive invariants and constructions that have been useful in the SFT case.

One can ask a similar question for injective and surjective CA. Note that if there were a universal surjective CA in the sense above, then the question of periodic points for surjective CA would reduce to answering it for the universal CA. Since the periodic points problem has been open for a long time, either there is no universal surjective CA, or else it will be hard to identify.