Clarification to "On the dimension of Furstenberg measure for $SL_2(\mathbb{R})$ random matrix products"

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1. There is a minor inconsistency in the way the proof of the main Theorem 1.1 is presented. Recall that Theorem 1.1 asserts that, under the hypotheses listed there, the Furstenberg measure ν satisfies

$$\dim \nu = \min\{1, h_{\mathrm{RW}}(\mu)/2\chi(\mu)\}$$

(see the paper for the notation and complete statement). In the proof of Proposition 4.9, on line -13, page 847, it is written "...assuming for the sake of contradiction that $\dim \nu < h_{\mathrm{RW}}(\mu)/2\chi$..." This is a contradiction only in the case when $h_{\mathrm{RW}}(\mu)/2\chi \leq 1$; otherwise the inequality holds trivially. The proof of Theorem 1.1 is continued on p. 865, Section 5.6. Here, on the 2nd line of Section 5.6 it is written "...fix a small $0 < \varepsilon < 1 - \dim \nu$..." It is here that we argue by contradiction with the assertion of Theorem 1.1 in the case when $h_{\mathrm{RW}}(\mu)/2\chi > 1$.

2. The proof of entropy porosity in Section 5.2 is incomplete. Specifically, at the end of Proposition 5.5, we conclude that $\frac{1}{m}H(\nu, \mathcal{D}_{i+m}|\mathcal{D}_i) > \alpha - \varepsilon'$ on average, as i ranges between 1 and n, but in the next equation, combined with a lower bound on the same entropies, we conclude that $\frac{1}{m}H(\nu_{x,i}, \mathcal{D}_{i+m}|\mathcal{D}_i) \leq \alpha + \varepsilon'$ with high probability. This does not follow, and the correct conclusion is just that $\frac{1}{m}H(\nu, \mathcal{D}_{i+m}|\mathcal{D}_i) \leq \alpha + \varepsilon'$ with high probability over i. To get porosity one must carry out a similar argument to give lower bounds on the component entropies, $\frac{1}{m}H(\nu_{x,i}, \mathcal{D}_{i+m}|\mathcal{D}_i)$. Similar analyses have been done e.g. in [16] in the proof of uniform entropy dimension, or in Section 3 of the forthcoming paper [BHR 2017].

References

[BHR 2017] Balázs Bárány, Michael Hochman, Ariel Rapaport, *Hausdorff dimension of planar self-affine sets and measures*, preprint, `https://arxiv.org/abs/1712.07353`.